Trigonometry I

BASIC TRIGONOMETRY IDENTITIES/RELATIONSHIPS

Complementary angle

$$\sin(90^o - x) = \cos x$$
$$\cos(90^o - x) = sinx$$
$$\tan(90^o - x) = \cot x$$
$$\cot(90^o - x) = tanx$$

Negative angle

$$\sin(-x) = \sin x$$
$$\cos(-x) = \cos x$$
$$\tan(-x) = -\tan x$$

Trigonometry relationship

$$\tan x = \frac{\sin x}{\cos x}$$

$$\cot x = \frac{1}{\tan x} = \frac{\cos x}{\sin x}$$

$$\sec x = \frac{1}{\cos x}$$

$$cosec\ x = \frac{1}{\sin x}$$

Trigonometry Identities

$$\sin^2 x + \cos^2 x = 1$$

$$1 + \tan^2 x = \sec^2 x$$

$$1 + \cot^2 x = cosec^2 x$$

Double Angle Formula

$$\sin 2x = 2\sin x \cos x$$

$$\begin{aligned}\cos 2x &= \cos^2 x - \sin^2 x \\ &= 2\cos^2 x - 1 \\ &= 1 - 2\sin^2 x\end{aligned}$$

$$\tan 2x = \frac{2\tan x}{1 - \tan^2 x}$$

RECAP: Trigonometry Proving

Trigonometry proving is a question that test a student's ability to observe trigonometry relationships, manipulating it until the LHS of the equation is equal to RHS of the equation. These questions are normally followed by the use of the proof to solve an equation.

Some **considerations** when performing trigonometry proving

1. The number of steps required is not more 7
2. Changing trigonometry function back to $\sin x$ or $\cos x$ before further manipulation (eg. Change $\sec x = \frac{1}{\cos x}$)
3. Use of trigonometry identities together with rules from quadratic equation identities

Eg.

Using $(a+b)^2 = a^2 + 2ab + b^2$

$$2\sin x \cos x + 1 = 0$$
$$2sinxcosx + \sin^2 x + \cos^2 x = 0$$
$$(\sin x + \cos x)^2 = 0$$

Using $a^2 - b^2 = (a+b)(a-b)$

$$\tan^2 x - 1 = (\tan x + 1)(\tan x - 1)$$

4. Combining fraction over fraction before simplification

$$\frac{1 - cosec^2 x}{3\cot x + cosec^2 x} = \frac{1 - \frac{1}{\sin^2 x}}{\frac{3\cos x}{\sin x} + \frac{1}{\sin^2 x}}$$

$$= \frac{\frac{\sin^2 x - 1}{\sin^2 x}}{\frac{3\cos x \sin x + 1}{\sin^2 x}}$$

$$= \frac{\sin^2 x - 1}{\sin^2 x} \times \frac{\sin^2 x}{3\cos x \sin x + 1}$$

$$= \frac{\sin^2 x - 1}{3\cos x \sin x + 1}$$

Try Question 1 and 2